

GUJARAT TECHNOLOGICAL UNIVERSITY**BE - SEMESTER-IV(OLD) – EXAMINATION – SUMMER 2019****Subject Code:140001****Date:09/05/2019****Subject Name: Mathematics-IV****Time:02:30 PM TO 05:30 PM****Total Marks: 70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

- Q.1 (a)** Find all roots of $\sqrt[3]{8i}$. 07
- (b)** 1) Find real and imaginary part of $f(z) = z^2 + 4z$. Also, calculate the value of f at $z = 1 + i$. 04
- 2) Show that $f(z) = \begin{cases} \frac{\text{Im}(z)}{|z|}; & z \neq 0 \\ 0; & z = 0 \end{cases}$ 03
- is not continuous at the origin.
- Q.2 (a)** Find the image of the region $|z| < 1$ under the transformation $w = 2z - i$. Sketch the region and its image. 07
- (b)** Show that $u(x, y) = 2x - x^3 + 3xy^2$ is harmonic in some domain D and find a harmonic conjugate of $u(x, y)$. 07
- OR**
- (b)** If $f(z)$ is an analytic function of z , show that 07
- $$\left(\frac{\partial}{\partial x} |f(z)|\right)^2 + \left(\frac{\partial}{\partial y} |f(z)|\right)^2 = |f'(z)|^2$$
- Q.3 (a)** Evaluate $\int_0^{2+i} z^2 dz$ along the line $y = x/2$ 07
- (b)** Evaluate: 07
1. $\oint \frac{z}{z-2} dz$, over the contour c , where c is the circle $|z| = 1$.
 2. $\oint \frac{z}{(1-z)^3} dz$, counterclockwise over C , where $C: |z| = 2$
 3. $\oint \frac{e^z}{(z-1)(z-3)} dz$, counterclockwise over C , where $C: |z| = 2$
- OR**
- Q.3 (a)** Determine the Laurent series expansion of $f(z) = \frac{1}{(z+1)(z+3)}$ valid for 07
- a) $|z| < 1$ b) $1 < |z| < 3$
- (b)** Using Newton's divided difference formula, compute $f(10.5)$ from the following data: 07
- | | | | | |
|-------|--------|--------|--------|--------|
| x: | 10 | 11 | 13 | 17 |
| f(x): | 2.3026 | 2.3979 | 2.5649 | 2.8332 |
- Q.4 (a)** Find a real root of the equation $x^3 + 4x^2 - 1 = 0$, lies between 0 and 1 by using bisection method correct to decimal places. 07
- (b)** Evaluate $\int_0^3 \frac{dx}{(1+x)^n}$ with $n=6$ by using Simpson's 3/8 rule and hence calculate $\ln 2$. 07

OR

- Q.4 (a) Solve the following system of equation using partial pivoting by Gauss Elimination method. **07**

$$8x_2 + 2x_3 = -7$$

$$3x_1 + 5x_2 + 2x_3 = 8$$

$$6x_1 + 2x_2 + 8x_3 = 26$$

- (b) Solve the following system of equations by using Gauss-Seidel method. **07**
 $10x + y + z = 6; \quad x + 10y + z = 6; \quad x + y + 10z = 6$

- Q.5 (a) Using the power method, find the largest eigenvalue of the matrix **07**

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

- (b) Apply Runge-Kutta fourth order method to find an approximation value of y **07**
when $x=0.1$ in step of 0.1 if $\frac{dy}{dx} = x + y^2, \quad y(0)=1$

OR

- Q.5 (a) Evaluate the integral $\int_0^1 \frac{dx}{(1+x)}$, by Gauss three point quadrature formula. **07**

- (b) Solve the differential equation $\frac{dy}{dx} + xy = 0; \quad y(0)=1$, from $x=0$ to $x=0.25$ using Euler's method taking step size 0.05. **07**

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